Time Synchronization Properties in OFDM System Based on Cyclic Prefix

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***Abstract*—The acceptable performance of the receiver side in communication systems can be achieved only if synchronization between the transmitter and the receiver exists. In this paper, two techniques that remove the influence of symbol time offset using cyclic prefix are analyzed to accomplish synchronization - accurate conduct of an orthogonal frequency division multiplexing (OFDM) system. The first technique is based on finding the maximum correlation between two blocks of samples and the second one is based on finding the similarity between two blocks, which is maximized when the difference between them is minimized. Two types of channel conditions are observed. The first one uses only additive white Gaussian noise, while the second one also uses the channel impulse response of the Rayleigh fading channel. By adding Rayleigh channel effect, both estimation performances, by calculating correlation and difference, deteriorate if sufficient level of signal-to-noise ratio and proper length of cyclic prefix are not provided.**

***Keywords*—Synchronization; OFDM; STO; Cyclic Prefix; Correlation; Difference; ISI**

I. Introduction

Higher transmission rates, managing large quantity of data and low-latency transmission are contemporary requirements of communication systems [1]. Traditionally, higher transmission rates can be achieved by increasing modulation order or by decreasing symbol duration. Higher modulation order implies larger number of symbols in the constellation diagram and thus magnifies the sensitivity to noise. Symbol duration cannot be reduced arbitrarily - reduction in time domain indicates expansion in frequency domain and possibly causes intersymbol interference (ISI). Orthogonal frequency division multiplexing (OFDM) is a multicarrier data transmission scheme which enables high transmission rates because it sends data on various subcarriers in parallel. To be specific, the available channel is split into adjacent narrow subchannels, i.e. subcarriers, whose frequencies are orthogonal [2]. Subchannels are multiplexed together - each one transmits certain different information at low rate, but the summed stream of data produces high rates.

OFDM is widely adopted in many wireless standards [3]; it has been a part of current 5G development and one of the potential candidates for 6G and beyond communication systems due to its flexibility in merging with other technologies and showing satisfying performance properties blended with machine and deep learning [1]. OFDM is highly valued because of its spectral efficiency (subchannels overlap without interference because of orthogonality) [1] and robust performance in environments with multipath fading [3]. It successfully prevents ISI by converting frequency-selective fading channels into several parallel flat-fading channels [4]. However, OFDM is not limitation-free. High peak-to-average power ratio (PAPR) and sensitivity to synchronization errors are the main difficulties [2]. The loss of orthogonality between the subcarriers, which can happen because of oscillator inconsistency in the transmitter and the receiver, is unacceptable in OFDM systems; it is called intercarrier interference (ICI). It causes severe performance degradation [5]. The purpose of the receiver - accurate signal detection and demodulation can be accomplished only if time and frequency synchronization between the transmitter and the receiver exists.

Multicarrier systems are more sensitive to time offsets than single-carrier systems [6], [7]. Symbol timing in OFDM cannot be found by selecting the most open part in the eye diagram, as in a single-carrier system; the estimation of the symbol start is required [5].

Symbol time offset (STO) and carrier frequency offset (CFO) estimation methods can be divided into two categories: data-aided methods, which require known pilot symbols and non-data-aided methods, which rely on guard interval, i.e. cyclic prefix (CP), make use of cyclostationarity of the OFDM signal, detect subcarrier symbol energy variations, etc. [8–10]. Recent research classifies data-aided methods into statistical based methods and deep-learning-based methods [4].

Since the non-data-aided methods that use conventional autocorrelation metric are subject to unsatisfactory behavior in case of striking multipath effect, a more robust transition metric-based method is proposed in [8]. The method does not require knowledge of the signal-to-noise ratio (SNR), the pulse shaping filter or the maximum channel delay spread. It enables the estimation of the starting point of the OFDM symbol using CP by tracking variations in time domain symbol energy (based on transition metric). The authors in [9] take interest in the enhancement of the method presented in [8] - they improve the energy transition metric to carry out better estimation and achieve higher throughput under low SNR conditions.

The authors in [6], [7] propose a joint, simultaneous maximum likelihood STO and CFO estimator while exploiting the structure of OFDM symbols (CP) rather than pilots. However, assumed channel conditions include only additive noise.

In 5G, more precisely in Internet of Vehicles technology, multipath fading and Doppler effect have a vast impact on the

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system [10]. For that reason, the authors in [10] propose a data-aided pilot based synchronization algorithm for filtered OFDM. A training sequence with real pseudorandom number (PN) pattern is created and sent to every subcarrier. The matching time-domain symbol, used for timing synchronization, manifests conjugate symmetry structure. CFO is estimated based on CP and the autocorrelation of PN sequence. The method outperforms conventional pilot-aided methods.

In [4], convolutional deep neural network STO estimation model is determined - it does not need pilots, known channel state information, CFO or any OFDM signal parameters. The authors examine the robustness of the method against the CFO.

In this paper, STO estimation in time domain using CP will be examined. In Section II, different cases of STO are explained and its effects counted. Section III gives mathematical description of the problem. Section IV analyzes the results. Section V concludes the paper.

II. Types of STO

There are four different cases of STO, comparing the estimated starting point and exact timing instance [11].

- The first case is when the estimated starting point coincides with the exact timing instance. Orthogonality is preserved and therefore OFDM symbol can be recovered successfully.
- In the second case, the estimated starting point is located before the exact timing instance, but after the end of maximum delay of the previous OFDM symbol. ISI has no influence here, but rotation around the origin in the constellation diagram happens due to phase offset.
- The third case implies that the estimated starting point overlaps with the channel response of the previous OFDM symbol, causing ISI and ICI.
- The final case is when the estimated starting point is after the exact timing instance (ISI and ICI also occur here).

In the paper, the second and fourth case will be analyzed. Considering 4-QAM modulation, constellation diagrams for STO with time delay of $\delta = \{3, -3, 2, -2\}$ samples are shown in Fig. 1. Constellation diagram of unaffected 4-QAM signal has 4 symbols, one in each quadrant. Fig. 1 shows imperfect constellation diagrams. The rotation of constellation diagram symbols, i.e. phase deviation can be seen in Fig. 1a and 1c. In Fig. 1b and 1d, both amplitude and phase distortion can been observed; the symbols are scattered. Rotation of symbols; phase distortion may be resolved by precise estimation and offset compensation by an equalizer. Symbol scattering is more complex - induced ISI needs sophisticated equalizers and advanced signal processing methods. An accurate STO estimation is required.

Although data-aided STO estimation is better than non-data-aided because it is not subject to multipath fading, the STO estimation using CP will be performed in this paper due to its simplicity - it does not require additional resources. Data-aided methods use training symbols - pilots, identical training sequences or sequences with a repeating structure. The similarity between two blocks of samples can be computed

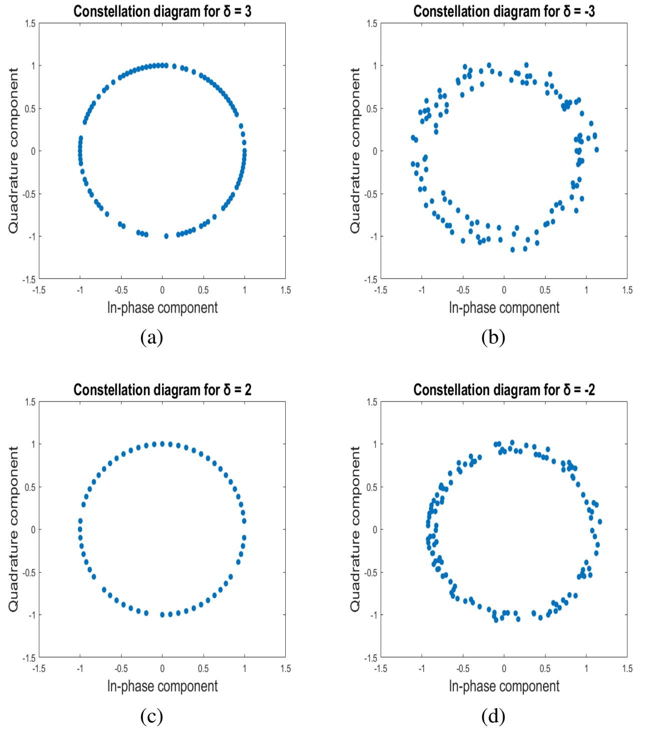

Figure 1. (a) Constellation diagram for $\delta = 3$, (b) constellation diagram for $\delta = -3$, (c) constellation diagram for $\delta = 2$, (d) constellation diagram for $\delta = -2$

using autocorrelation feature of the training signal [11]. Non-data-aided methods which use CP exploit the overlap between samples of CP and OFDM useful data samples. Two sliding windows are used for estimating this similarity [11]. The procedure can be seen in Fig. 2. Windows $W1$ and $W2$ both have size of N_G samples. The useful part of OFDM symbol has a duration of T_{sub}. CP lasts for T_G. Both windows save samples which they encounter. $W1$ slides along CP, while $W2$ slides along the useful part of OFDM symbol. The samples are compared in every position. The correlation between blocks, i.e. samples located within windows, can be calculated [11]. The idea is to maximize the correlation between two blocks. The difference between blocks may also be calculated [11]. It needs to be minimal in order for the similarity of the samples to be maximal.

III. Mathematical Description of the Problem

Both the correlation-based method (CBM) and the difference-based method (DBM) will be used for the correct

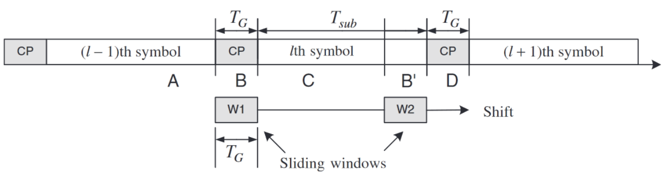

Figure 2. STO estimation with two sliding windows [11]

detection of STO using CP in this paper. Since CP is a copy of a part of the symbol (most often the end), this implies that we could look for similarities for estimation. We will use blocks of the same length as CP (N samples over T seconds). The blocks can move, to find the similarity between the samples. Maximum similarity is obtained when the CP of OFDM symbol is found in the first moving block.

The mathematical formulation of CBM is given by [11]:

$$\hat{\delta} = \underset{\delta}{\operatorname{argmax}} \sum_{i=\delta}^{N_G-1+\delta} y_l[n+i] * y_l^*[n+N+i]. \quad (1)$$

DBM finds STO by searching the point where the squared difference between two blocks is minimized (to eliminate CFO influence). The mathematical formulation is given by [11]:

$$\hat{\delta} = \underset{\delta}{\operatorname{argmin}} \sum_{i=\delta}^{N_G-1+\delta} |y_l[n+i] - y_l^*[n+N+i]|^2 . \quad (2)$$

In the above equations, l-th OFDM symbol is considered. Label $\hat{\delta}$ represents the estimated time delay, δ the actual time delay, N_G the number of samples in CP, $y_l[n+i]$ the received signal at time $n+i$, $y_l^*[n+N+i]$ the complex conjugate of the received signal at time $n+N+i$, and N the number of symbols transmitted in the signal.

IV. Results and Discussion

The code for the analysis of STO estimation using CP is implemented by script written in MATLAB. In the first case, generated OFDM symbols will be sent through the channel with additive white Gaussian noise (AWGN). However, the existence of only white noise in the channel is an unrealistic case. For that reason, in the second case, Rayleigh channel effects will be included. Rayleigh channel will be modeled using channel impulse response (CIR). Impulse response is obtained by passing Dirac delta function through the channel. Complex coefficients of CIR are extracted from the new, distorted channel characteristic acquired at the receiver. Rayleigh fading channel model is treated as Gaussian random variable with zero mean and unit variance. Because of that, CIR consists of 10 coefficients which are different every time the code is being run (because of using rand function). The values of the coefficients are respectively [-0.2338 + 0.1770i, 0.1573 - 0.0179i, 0.1352 + 0.1641i, -0.1318 - 0.2919i, -0.1715 + 0.3104i, 0.5049 - 0.1209i, 0.2021 - 0.6263i, 0.0621 + 0.1324i, 0.1568 - 0.0362i, 0.0113 - 0.0004i]. In theory, such channels should degrade the performances of our estimation algorithms. Rayleigh fading contributes to worsening of communication system performance in general: signal weakening, ISI, frequency selectivity and Doppler spread. Magnitude and phase response of the modeled Rayleigh channel are shown in Fig. 3. Magnitude response shows the gain or attenuation of the filter at each frequency, while the phase response shows the amount of phase shift introduced by the filter at each frequency.

In this paper, we will ignore the effects of CFO. Results will be shown depending on the level of the signal-to-noise (SNR) ratio and the length of CP. The behavior of the system will be

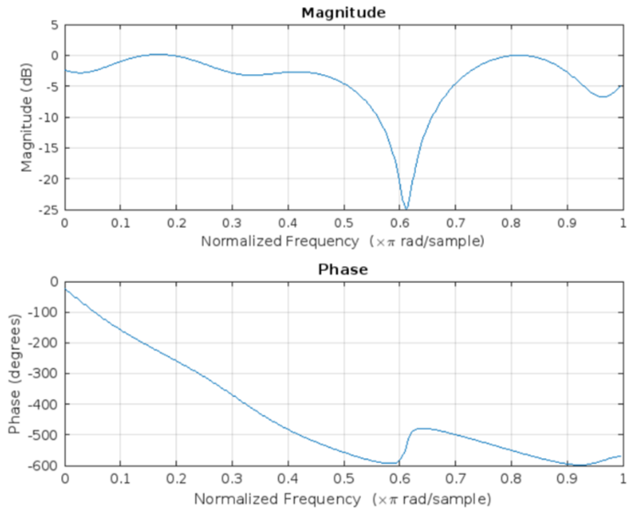

Figure 3. Magnitude and phase response of complex-valued filter

observed in cases when SNR is 2 dB, 4 dB, 6 dB, 8 dB and 10 dB. The idea is to gradually notice changes in the estimated STO values with slight SNR changes. SNR value of 10 dB provides good transmission conditions, while SNR value of 2 dB makes transmission system very sensitive to deterioration. The length of CP will amount to 32 and 16 samples. The length of CP of 32 samples provides good synchronization conditions while the length of CP of 16 samples makes the system sensitive to synchronization errors. STO with time delay of $\delta = \{3, -3, 2, -2\}$ samples will be analyzed.

In Table I, results of the estimated STO for CBM and DBM in AWGN channel are shown. In Table II, the results of the estimated STO for CBM and DBM in Rayleigh channel are shown. Graphical representation of the results can be seen in [12].

In AWGN channel, by increasing SNR, the improvement of the estimated STO value can be seen up to a certain point. E.g., there is no difference in STO estimation given the SNR value of 8 and 10 dB, while minimal differences can be noticed with the SNR equal to 6 dB (while CP is 32 samples). Further increase of SNR would not lead to any improvement of STO estimation. When the value of SNR is reduced to 2 dB, the estimated STO can no longer give reliable information where the offset really happened. On the other hand, when the value of SNR is 10 dB, the algorithm works fine and can detect where STO happened. If the length of CP is decreased on 16 samples, unsatisfactory results are obtained even sooner for the same values of SNR.

When Rayleigh channel effects are added, the error increases, which means that the efficiency of the estimation algorithms significantly drops. Neither CBM nor DBM are able to make a correct guess of STO, even in the case with the highest SNR (10 dB) and bigger CP (32 samples). When CP is equal to 16 samples, tremendous deviations of CBM can be seen even for SNR equal to 10 dB: $\{16, -31, 5, 69\}$.

TABLE I. Estimated STO in AWGN channel

		CP = 32		CP = 16	
SNR	Real STO	CBM STO	DBM STO	CBM STO	DBM STO
2 dB	3	2	2	10	−4
	−3	−3	−3	−3	3
	2	2	4	2	3
	−2	1	−2	−3	1
4 dB	3	2	2	2	−4
	−3	−3	−3	−3	3
	2	2	4	2	2
	−2	1	−2	−3	1
6 dB	3	2	2	2	−4
	−3	−3	−3	−3	3
	2	2	1	2	2
	−2	−1	−2	−3	1
8 dB	3	2	2	2	3
	−3	−3	−3	−3	−3
	2	2	2	2	2
	−2	−1	−2	−2	−2
10 dB	3	2	2	2	3
	−3	−3	−3	−3	−3
	2	2	2	2	2
	−2	−1	−2	−2	−2

TABLE II. Estimated STO in Rayleigh channel

		CP = 32		CP = 16	
SNR	Real STO	CBM STO	DBM STO	CBM STO	DBM STO
2 dB	3	9	7	16	1
	−3	3	54	−31	4
	2	8	12	5	8
	−2	4	−4	69	3
4 dB	3	9	7	16	1
	−3	3	1	−31	3
	2	8	7	5	8
	−2	4	−4	69	3
6 dB	3	9	7	16	1
	−3	3	1	−31	3
	2	8	7	5	8
	−2	4	−3	69	3
8 dB	3	9	7	16	5
	−3	3	1	−31	3
	2	6	7	5	8
	−2	4	−3	69	3
10 dB	3	9	7	16	5
	−3	3	1	−31	3
	2	6	7	5	8
	−2	4	−3	69	4

V. Conclusion

Synchronization plays a key role in ensuring successful functioning of OFDM systems. OFDM systems endure time and frequency offset problems (STO and CFO). In this paper, estimation of STO has been presented for OFDM system using CP. Two methods were analyzed - correlation-based and difference-based. The effects of the methods were observed depending on the length of CP and the level of SNR in AWGN channel and in Rayleigh channel. By decreasing SNR level, it gets progressively more difficult to estimate where STO happened. Results of both analyzed methods show satisfactory level of STO estimation in AWGN channel. Additionally, the results indicate frailty of estimation when Rayleigh channel effect is added; both estimation by correlation and difference are ineffective if a sufficient level of SNR and the appropriate length of CP is not provided. The only way to eliminate this problem is to always ensure a sufficient level of SNR and the appropriate length of CP or use more complex techniques.

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References

[1] F. H. Juwono and R. Reine, “Future OFDM-based communication systems towards 6G and beyond: machine learning approaches,” *Green Intelligent Systems and Applications*, vol. 1, 2021.

[2] C. D. Parekha and J. M. Patel, “Overview on synchronization in OFDM systems,” in *International Conference on Advances in Computing, Communication, Automation*, pp. 1–6, 2016.

[3] T. Hwang, C. Yang, S. Li, and G. Ye Li, “OFDM and its wireless applications: A survey,” *IEEE Transactions on Vehicular Technology*, vol. 58, no. 4, pp. 1673–1694, 2009.

[4] M. S. Chaudhari and S. Majhi, “STO estimation for OFDM system using CDM,” *IEEE Communications Letters*, vol. 26, no. 11, pp. 2651–2655, 2022.

[5] T. M. Schmidl and D. C. Cox, “Robust frequency and timing synchronization for OFDM,” *IEEE Transactions on Communications*, vol. 45, no. 12, pp. 1613–1621, 1997.

[6] J.-J. van de Beck, M. Sandell, and P. O. Börjesson, “On synchronization in OFDM systems using the cyclic prefix,” in *Radio Vetenskaplig Konferens*, pp. 663–667, 1996.

[7] J.-J. van de Beck, M. Sandell, and P. O. Börjesson, “ML estimation of time and frequency offset in OFDM systems,” *IEEE Transactions on Signal Processing*, vol. 45, no. 7, pp. 1800–1805, 1997.

[8] V. Le Nir, T. van Waterschoot, J. Duplicy, and M. Moonen, “Blind coarse timing offset estimation for CP-OFDM and ZP-OFDM transmission over frequency selective channels,” *EURASIP Journal on Wireless Communications and Networking*, vol. 262813(2010), 2009.

[9] Q. Wang, M. Simko, and M. Rupp, “Modified symbol timing offset estimation for OFDM over frequency selective channels,” in *IEEE Vehicular Technology Conference*, pp. 1–5, 2011.

[10] X. Feng, M. Wang, L. Chen, *et al.*, “Blind coarse timing offset estimation for CP-OFDM and ZP-OFDM transmission over frequency selective channels,” *EURASIP Journal on Advances in Signal Processing*, vol. 38(2020), 2020.

[11] Y. S. Cho, J. Kim, W. Y. Yang, and C. G. Kang, *MIMO-OFDM Wireless Communications with MATLAB*. John Wiley & Sons (Asia) Pte Ltd, 2010.

[12] I. Cavlek and S. Grgic, “Estimation of symbol time offset using cyclic prefix in OFDM system.” https://arxiv.org/abs/2305.02876, 2023.